From 'Why DMFT?' to 'Why not DMFT!'

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Quasiparticle Electronic Structure and Augmented LMTOs

MOST RECENT ARTICLE Spin-orbit Torgues in CoPt Multilayers October 30, 2018 | PAPERS · NONEQUILIBRIUM GREEN'S FUNCTIONS



We have demonstrated the feasibility of calculating the spin-orbit torques in layered systems within density-functional theory, augmented by an Anderson model to treat disorder. Terms beyond the usual damping-like and field-like torques were found. While the torques that contribute to damping are almost entirely due to spin-orbit coupling on the Pt atoms, the fieldlike torque does not require it.

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About Questaal

Questaal is a suite of programs for electronic structure simulations.

What are materials made of?

Around 2600 years back: Kanada and Democritus

ATOMS!

Space, time!

From Electrons to Atoms!

Walter Kohn, Landau : electrons!

Mott, Peierls, Van Vleck, Anderson, Hubbard and Chemists know it!

When John Hubbard started his career in theoretical physics it was known that a surprisingly good understanding of the electron gas in metals could be obtained by ignoring the Coulomb interaction among the electrons but there was no understanding of how to develop a consistent way to treat these interactions. John Hubbard's doctoral the-

T M Rice, 1980

From Electrons to Atoms: continued

While the importance of correlation in causing the breakdown of band theory and insulating character of magnetic insulators was known from the work of Mott, Peierls, Van Vleck and Anderson, it was John Hubbard who put the problem on a firm foundation. The famous Hubbard Hamiltonian for electron correlation is as crucial and fundamental as the Ising and Heisenberg Hamiltonian for localized spins and by now has spawned almost as much work. However the large literature on the Hubbard Hamiltonian that now exists also serves to emphasize the importan ce of his original contribution and the depth of his understanding. W.Kohn has described his contribution as "the basis of much of our present thinking about the electronic structure of large classes of metals and insulators". It is also the basis of much of what we are

T M Rice, 1980

Atomic correlations matter! How obvious is it?

From Electrons to Atoms: From Fermi gas to atomic multiplets

For most T and P: quasi-particles (Thanks to Lev Landau) are long lived

Large Fermi energy/quasi-particle coherence scale

From Electrons to Atoms: From Fermi gas to atomic multiplets



No Fermi energy/quasi-particle coherence scale -Atomic Multiplets

Most materials are in-between these two-extremes : small enough quasi-particle coherence scale and atomic like excitations

From Fermi gas to atomic multiplets : Do we see them?



From Fermi gas to atomic multiplets : Do we see them?



From Fermi gas to atomic multiplets : Low energy and high energy !



matters in metals too

From Fermi gas to atomic multiplets : Low energy and high energy !



Fujimori et al, PRL 69, 1796 (1992)



Sekiyama et al., PRL 93, 156402 (2004)

Low energy quasiparticle coherence scale ($z = m/m^*$) needs to be rescaled by a factor of 2 from band theory calculations ---one would imagine it could driven to 0 by continuosly making it narrower

Theory for explaining high and low energy physics

The Brinkman-Rice transition



P Fazekas, Notes on Electronic Correlation and Magnetism

U/D

But then what happens to the rest of the spectral weight? Where do they go?

Theory for explaining high and low energy physics

The Brinkman-Rice transition

Correlated metallic state. Fermi liquid like aproach



2D 8. U/D=0 ь U/D=1 с U/D=2.4 QP UHB LHB d U/D=4 -U/2 $0 = E_{F}$ +U/2 Excitation energy

But then what happens to the rest of the spectral weight? Where do they go?

P Fazekas, Notes on Electronic Correlation and Magnetism

Theory for explaining high and low energy physics

is lowered. This is what Dynamical Mean-Field Theory (DMFT) does. The term *dynamical* is perhaps not ideally chosen, since we are not talking here about the out-of-equilibrium dynamics of the system. Instead, it indicates that the theory handles the different time-scales or energy-scales involved in the excitation spectrum of the system at equilibrium. In order to do so, DMFT introduces a generalization of the classical Weiss mean-field concept to that of a full function of energy (or time scale).

A Georges

E. Pavarini, E. Koch, D. Vollhardt, and A. Lichtenstein DMFT at 25: Infinite Dimensions Modeling and Simulation Vol. 4 Forschungszentrum Jülich, 2014, ISBN 978-3-89336-953-9 http://www.cond-mat.de/events/correl14



But then what happens to the rest of the spectral weight? Where do they go?

DMFT: How does it work?



$$G_{imp}^{(i)} = \frac{1}{i\omega + \mu - \Delta - \Xi(\omega)}$$

$$G(\mu, i\omega) = \frac{1}{i\omega + \mu - \varepsilon_{k} - \Xi(\bar{\mu}, i\omega)}$$

$$G_{imp}^{(i)} = \sum G(\mu, i\omega)$$

$$G_{\rm imp}[i\omega_n,\Delta] = \sum_{\mathbf{k}} \frac{1}{G_{\rm imp}[i\omega_n;\Delta]^{-1} + \Delta(i\omega_n) - \varepsilon_{\mathbf{k}}}$$

Fig. 6: The Dynamical Mean-Field Theory (DMFT) concept. A solid is viewed as an arra atoms exchanging electrons, rather than as a gas of interacting electrons moving in an periodic potential. DMFT replaces the solid by a single atom exchanging electrons with a self-consistent medium and takes into account local many-body correlations on each site.

DMFT: When does it work?

The single-site DMFT construction becomes exact in the following limits.

- In the atomic limit $t_{ij} = 0$, by construction (then, $\Delta = 0$).
- In the non-interacting limit U = 0. Indeed, in this case the self-energy $\Sigma = 0$, so that it is trivially k-independent.
- Hence, both the limit of a non-interacting band and that of isolated atoms are correctly reproduced by DMFT, which provides and interpolating scheme between these extreme cases.
- In the limit of infinite lattice coordination (infinite number of spatial dimensions), first introduced for fermions in the pioneering work of Metzner and Vollhardt [23]. The hopping must be scaled as $t_{ij} = t/\sqrt{d}$ for this limit to be properly defined and non-trivial.
- Being an exact solution of Hubbard-like models in the limit of infinite dimensions, it is thus guaranteed that DMFT preserves all sum-rules and conservation laws.

A Georges, DMFT at 25

DMFT: Let's simplify one step further

TABLE 1. Correspondance between the mean-field theory of a classical system and the dynamical mean-field theory of a quantum system.

| Quantum Case | Classical Case | |
|---|--|--------------------------------------|
| $-\sum_{ij\sigma} t_{ij}c_{i\sigma}^+ c_{j\sigma} + \sum_i H_{atom}(i)$ | $ H = -\sum_{(ij)} J_{ij} S_i S_j - h \sum_i S_i$ | Hamiltonian |
| $G_{ii}(i\omega_n) = - < c^+_i(i\omega_n) c_i(i\omega_n) >$ | $ m_i = < S_i >$ | Local Observable |
| $ \begin{vmatrix} H_{eff} = H_{atom} + \sum_{l\sigma} \widetilde{\epsilon}_l a_{l\sigma}^+ a_{l\sigma} + \\ + \sum_{l\sigma} V_l (a_{l\sigma}^+ c_{\sigma} + h.c) \end{vmatrix} $ | $H_{eff} = -h_{eff}S$ | Effective single-site Hamiltonian |
| $igg \Delta(i \omega_n) = \sum_l rac{ V_l ^2}{i \omega_n - \widetilde{arepsilon_l}} \ \mathscr{G}_0^{-1}(i \omega_n) \equiv i \omega_n + \mu - \Delta(i \omega_n)$ | h_{eff} | Weiss function/Weiss field |
| $\boxed{\sum_{\mathbf{k}} [\Delta(i\omega_n) + G(i\omega_n)^{-1} - \varepsilon_{\mathbf{k}}]^{-1} = G(i\omega_n)}$ | $ig \qquad h_{eff} = \sum_j J_{ij} m_j + h$ | Self-consistency relation |

TABLE 2. Comparison of theories based on functionals of a local observable

| Theory | MFT | DFT | DMFT | |
|----------------------------|---------------------------|----------------------------------|----------------------------|--|
| Quantity | Local magnetization m_i | Local density $n(x)$ | Local GF $G_{ii}(\omega)$ | |
| Equivalent system | Spin in effective field | Electrons in effective potential | Quantum impurity model | |
| Generalised Weiss field | Effective local field | Kohn-Sham potential | Effective hybridisation | |

A Georges, arxiv 2004

DMFT in our implementation

Need to build the impurity Hamiltonian and the hybridization of the impurity with the bath



DMFT Self Consistency: in our implementations



(External Charge self consistency/ Self energy)

Systems with $T_N < \Delta_q$: Magnetic ordering is not source of Metal-Insulator transition

$$La_2CuO_4$$
: $T_N = 312 \text{ K} (26 \text{ meV}), \Delta_g = 2.1 \text{ eV}$

YTiO₃ :
$$T_c = 30 \text{ K} (3 \text{ meV}), \Delta_g = 1.0 \text{ eV}$$

LaTiO₃ :
$$T_N = 140 \text{ K} (12 \text{ meV}), \Delta_g = 0.2 \text{ eV}$$

 $\Delta_{\alpha} > T > J$ these systems are paramagnetic insulator

----- Don't need magnetic ordering to become insulator ----- Mott insulators ----- charge blocking---suppress double occupancy



Single and two-particle response in La₂CuO₄





| Table | | | | | | | | |
|--|------|---|---|---------------------|---|------------------|----------------|------------|
| Compound | Ζ | $\begin{array}{cc} \Gamma & \mathrm{in} \\ \mathrm{eV} \end{array}$ | $\begin{array}{cc} \Delta_x & \mathrm{in} \\ \mathrm{eV} & \end{array}$ | T_c^{max} in K | $\begin{array}{cc} \Delta & \mathrm{in} \\ \mathrm{eV} \end{array}$ | Cu $d_{x^2-y^2}$ | Axial orbitals | $O-p_{xy}$ |
| $CO \ (\delta = 0) \ QSGW$ | 0.00 | 11.4 | -1.53 | 34 | 0.78 | 0.5437 | 0.108 | 0.2630 |
| CO ($\delta = 0.03$) QSGW | 0.02 | 0.41 | -1.49 | 79 | 1.0 | 0.5454 | 0.105 | 0.2615 |
| CO ($\delta = 0.05$) QSGW | 0.08 | 10^{-4} | -1.45 | 53 | 0 | 0.5471 | 0.092 | 0.2607 |
| $CO(\delta = 0.09) QSGW$ | 0.41 | 10^{-9} | -1.37 | 11 | 0 | 0.5507 | 0.086 | 0.2591 |
| $LCO(\delta = 0)$ LDA | | | | | | 0.40067 | 0.22 | 0.2798 |
| $\operatorname{ICO}\operatorname{QS} GW$ | 0.44 | 10^{-6} | -0.97 | 25 | 0 | 0.63007 | 0.033 | 0.2520 |

Swagata Acharya et al., Phys. Rev. X 8, 021038 (2018)

Cu-d², O-p, and Cu-4s

Cu-d², O-p, and Cu-4s

QSGW

are suppressed significantly in

are huge in LDA

Hyowon Park PhD thesis, Rutgers Lib. (2012)

Single and two-particle response in La₂CuO₄



(d) Single-particle response in doped La_{2-x}Sr CuO4





X=0.06

X=0.02



doping



Single and two-particle response in La_{1.88}Sr_{0.12}CuO₄







h in (h,0.5,0)

h in (h,0.5,0)





Experimental Phase Diagram : Ca_{2-x}Sr_xRuO₄



Carlo et al., Nature (2013)

S. Nakatsuji et al. PRB (2000).



Outstanding issues at x = 2.0

Incoherence-coherence crossover at 25 K
 Resistivity crosses from ~T to ~T²

Susceptibility crosses from T $^{-1}$ (Curie-Weiss) to T⁰ (Pauli)

S. Acharya et al. Scientific Reports 7, 43033 (2017)

Instabilities of a FL : triplet unconventional superconductivity (below 1.5 K)



Single-particle Response of Sr₂RuO₄



S. Acharya et al. Scientific Reports 7, 43033 (2017)

41 K : Scattering rate ~T to ~T² crossover
30 K: Resistivity ~T to ~T² crossover (Maeno et al. 2004)

Thermal Scaling of Local Spin Susceptibilities



Two-particle Response: Sr₂RuO₄



Local moment quenches around 25 K Charge susceptibility is singular down to 12 K

Phase diagram for the iso-electronic series Ca_{2-x}Sr_xRuO₄



Local Static Spin Susceptibilities: x = 0, 0.5, 2.0

X=0.5

S.Acharya et al. JPC, 2, 075004 (2018),



Scattering rates: FL and NFL



Uniform Spin Susceptibilities: x = 2.0, 0.5, 0.0



Spin susceptibility



Inelastic neutron scattering shows strong peaks at incommensurate $q = q^* = (0.3, 0.3, 0)$ with $\omega_{max} \approx 10 \text{ meV}$.

QSGW+DMFT result for χ^{s} is nearly identical. Peaks do not derive from the nesting as in Cr: proper treatment of vertex is essential!



P. Steffens et al, arXiv:1808.05855

arXiv:1811.05143

Valence fluctuations: Atomic Multiplets

Plutonium is in ideall f⁵ state and curium is f⁷ state



Shim, Kristjan, Gabi, Nature volume446, pages513-516 (29 March 2007)



What you don't expect from DMFT



Single-site DMFT does not have --Momentum dependent incoherence---Im $\Sigma(w)$ ---- one needs momentum dependent life time effect $-\text{Im}\Sigma(k,w)$ ----Dual Fermions, Cluster DMFT. Diagrammatic QMC